Comparing the first and second rates gives $v_1 \simeq \delta_2^{-1/6}(Lk/\lambda(T))^{1/3}v_{Ta}$. The fact that $v_1$ must clearly satisfy the inequality $\lambda(v_1)/k < L$ implies that $f_a$ is close to the Maxwellian function in almost the entire region $\lambda(\varepsilon)/k \leqslant L$ for concentrations $n_Z < (Lk/\lambda(T))^{1/2}n_a/Z^2$.

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# NUMERICAL EVALUATION OF DIODE GAP BRIDGING BY IONS IN A PLANAR DIODE


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UDC 533.932.12


To obtain powerful electron beams the so-called planar diodes, in which the cathode as well as the anode are disks of radius R exceeding considerably the gap $a$ between the electrodes, are often used. If the self-magnetic field of the beam can be ignored (for example, when the diode is in a strong external magnetic field), then the motion of the electrons in the diode is one-dimensional. The problem of determining the stationary current passing through such diode in the nonrelativistic case is easily solved, the corresponding dependence being given by the well-known "3/2 rule." This solution can be extended to relativistic potentials [1]; moreover, the ion emission from the anode can also be included [2]. It is assumed again that the diode current is time-independent. In this article results of numerical computations of the nonstationary electron-diode operation in a state in which the diode gap is filled by ions emitted from the anode are given. The case of ion emission of one type, as well as the cases in which ions are emitted with different masses, is studied.

## 1. Formulation of the Problem

The anode plasma which arises as a result of diode operation is a source of ions emitted into the diode gap. The distinctive time scale $\tau$ of the problem is the duration of ion transit between the electrodes $a$, $\tau \sim a(M/e\varphi_0)^{1/2}$, where $\varphi_0$ is the potential difference between the cathode and the anode; M is the ion mass (ions are regarded as singly charged). It is of interest to analyze the times $t \leqslant \tau$ when the problem is essentially nonstationary (the time t is counted from the start of the ion emission). For $t \gg \tau$ the diode operates in a stationary manner with an incoming flow of electrons and ions; in the latter case the results of [2] become applicable. It is assumed that in the period of time which is short compared with $\tau$ a sufficiently dense plasma


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is formed so that the emitted ion current is restricted by the space charge. Formally, this means that the electric field at the anode surface must vanish.

The smallness of the parameter (m/M) results in a considerable simplification of the solution (m is the electron mass). Since the crossing time of the electron gap $\tau_e \sim a(m/e\varphi_0)^{1/2}$ is small compared with $\tau$, the effects of no stationarity as regards the electrons can be ignored. The latter enables one, in particular, to express the electron density $n_e(z, t)$ by means of the potential $\varphi(z, t)$ at the point z:

$$n_e = (j_e/ce)\,[1 - (1 + e\varphi/mc^2)^{-2}]^{-1/2}, \tag{1.1}$$

where $j_e = j_e(t)$ is the electron-current density; c is the light velocity. The coordinate z is measured from the cathode and the cathode potential is assumed to vanish.

The density of the ions $n_i$ and their velocity v can be determined from the hydrodynamic equations:

$$\partial n_i/\partial t + \partial n_i v/\partial z = 0; \tag{1.2}$$

$$\partial v/\partial t + v\partial v/\partial z = -\,\partial\varphi/\partial z, \tag{1.3}$$

the potential $\varphi$ satisfying the Poisson equation, which can be written with the aid of (1.1) as

$$\partial^2\varphi/\partial z^2 = j_e\{1 - [1 + (\gamma - 1)\varphi]^{-2}\}^{-1/2} - n_i. \tag{1.4}$$

Here, as well as in our further considerations, the following dimensionless quantities are employed:

$$\varphi \to \varphi/\varphi_0,\ z \to z/a,\ t \to (t/a)\sqrt{e\varphi_0/M},\ v \to v\sqrt{M/e\varphi_0},$$
$$n_{e,i} \to n_{e,i}4\pi e a^2/\varphi_0,\ j_e \to j_e 4\pi a^2/c\varphi_0,\ \gamma = 1 + e\varphi_0/mc^2.$$

The boundary conditions for the system (1.2)-(1.4) are given by

$$\varphi|_{z=0} = 0,\ \partial\varphi/\partial z|_{z=0} = 0; \tag{1.5}$$

$$\partial\varphi/\partial z|_{z=1} = 0; \tag{1.6}$$

$$\varphi|_{z=1} = 1. \tag{1.7}$$

The requirement that the electric field vanishes at the points z = 0, 1 corresponds to the assumption that the cathode and the anode have infinite emission power.

If the anode plasma contains ions with differing masses, then they will be emitted into the diode gap in the ratio determined by the component composition of the anode plasma. Side by side with the case of single-component plasma the situation when the plasma consists of ions of two kinds with the mass ratio $\mu$ and the relative concentration $\alpha$ of heavy ions is also considered. In this case Eqs. (1.2) and (1.3) must be written for each component. From the computational point of view it does not matter whether one takes into account yet another kind of ions; therefore, when describing the numerical method only the variant of a single-component plasma is considered.

It is noticed that in the nonrelativistic limit ($\gamma - 1 \ll 1$) the parameter $\gamma$ can be eliminated from the system (1.2)-(1.4) by modifying the unit of measurement $j_e$. The system of equations thus obtained is of universal form, that is, it contains no unknown parameters.

## 2. Description of the Numerical Method

The system of equations (1.2)-(1.4) was solved numerically on an electronic computer with the aid of a modified method of the particles in cells [3]. According to this method, counting particles which simulate a planar layer of ions moving as one entity are introduced. Such a particle is characterized by the jump magnitude $\Delta E$ which tests the electrical field when passing through the ion layer. The equations of motion for a particle i are

$$dv_i/dt = E_i(t),\ dz_i/dt = v_i,$$

where $E_i(t) = E(Z_i(t), t)$ is approximated by a finite-difference scheme centered in time of the second order of accuracy [3]

$$z_i(t) = z_i(t - \Delta t) + \Delta t v_i\left(t - \tfrac{1}{2}\Delta t\right),\ v_i\left(t + \tfrac{1}{2}\Delta t\right) = v_i\left(t - \tfrac{1}{2}\Delta t\right) + \Delta t E_i(t), \tag{2.1}$$

where $\Delta t$ is the temporal step.

At the initial-time instant (the instant of anode plasma forming) at the point $z = 1$ there are N counting particles. The electron current $j_{e0}$ when $t = 0$ and the density $n_e(z, 0)$ are found by ignoring ions in (1.1), (1.4), (1.5), and (1.7). The magnitude $\Delta E$ is determined by the condition that the arising ions screen the electrical field on the anode [condition (1.6)]; since the field on the cathode also vanishes,

$$\Delta E = \frac{1}{N} \int_0^1 n_e(z, 0)\, dz,$$

The subsequent computations progress in the following order. To solve Eq. (1.4) numerically one subdivides the interval $0 \le z \le 1$ into M cells. Knowing the distribution of the counting particles one determines the ion density in each cell. Equation (1.4) with the initial conditions (1.5) is solved, and one also determines by enumeration a value of $j_e$ such that the condition (1.7) is also satisfied. The particles travel through one temporal step in agreement with (2.1), those escaping beyond the limit of the interval $0 \le z \le 1$ being excluded from our considerations. Finally, one adds (or subtracts) to the last cell as many particles as it is required to make $E|_{z=1}$ differ from zero by not more than $\Delta E$ [the condition (1.6) is satisfied with this error]. Then the entire procedure is repeated.

The field $E_i(t)$ acting on the particle i is determined as follows. One represents it as a sum of the electron field and the ion field at the point $z_i$. The latter is computed exactly (it is equal to the product of $\Delta E$ and the number of particles between the particle i and the cathode). The electron field is found by interpolating between the values at the nearest nodes of a space grid.

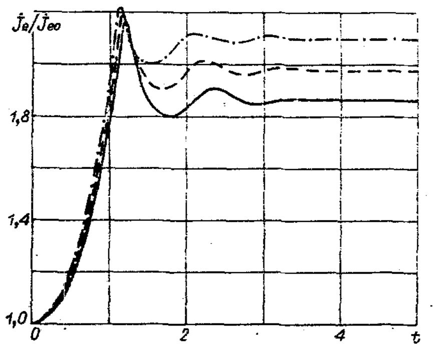


Fig. 1

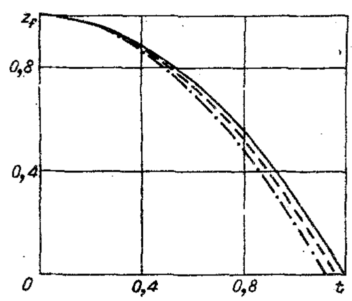


Fig. 2

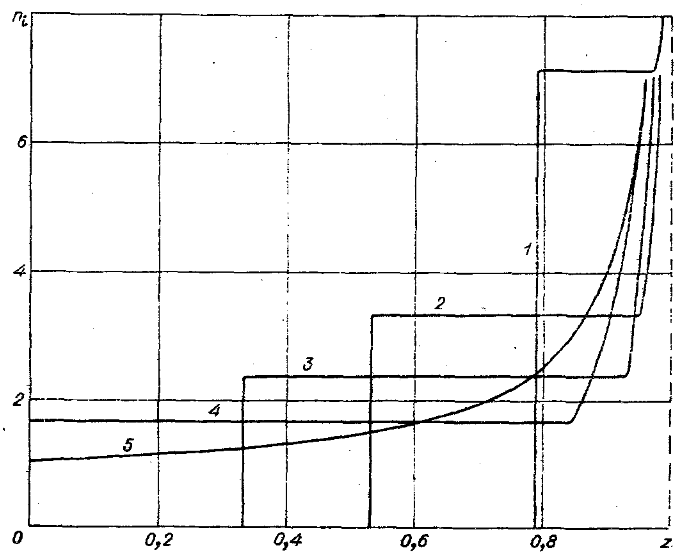


Fig. 3

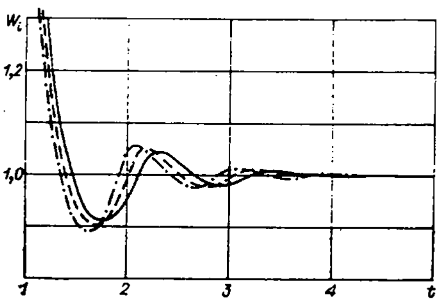


Fig. 4

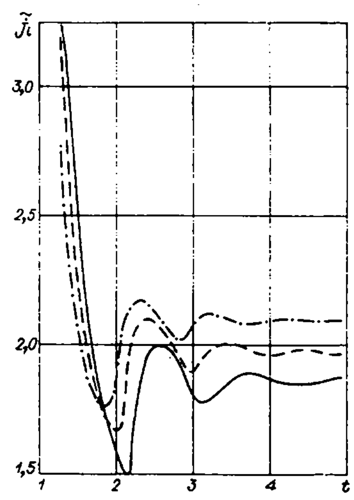


Fig. 5

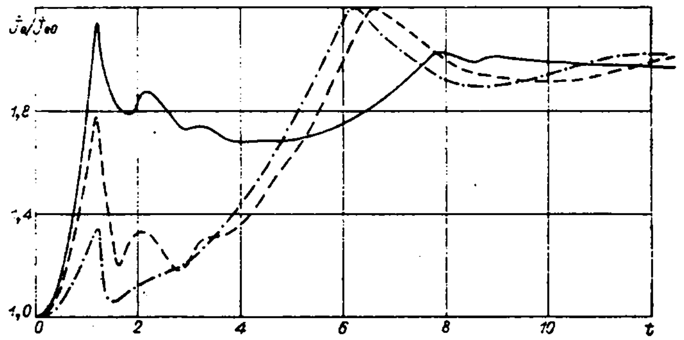


Fig. 6

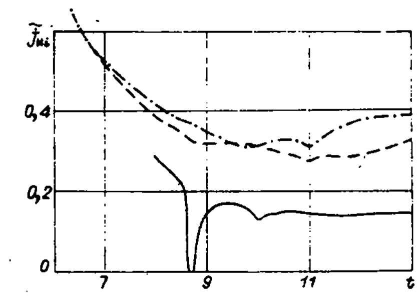


Fig. 7

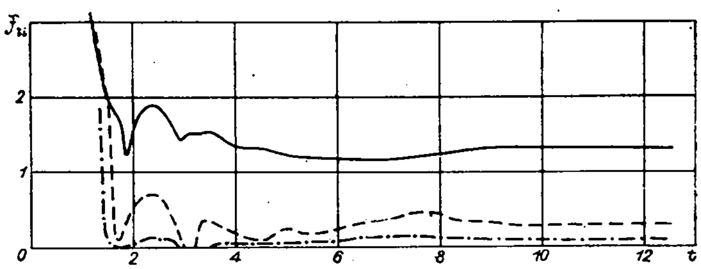


Fig. 8

The validity of the computations was verified by modifying the computation parameters $\Delta t$, M, N, which have no effect on the physical formulation of the problem. Having modified them two to three times it was found that the results varied within 1%. The typical values were $\Delta t_{,}=0.01$, $M=200$, $N=1200$.

## 3. Computation Results and Their Discussion

The main results for a single-component plasma are shown in Figs. 1-5. Three variants were evaluated with different values of $\gamma$: The solid line corresponds to $\gamma=1.05$; the dashed line, to $\gamma=3$; and the dashed-dot line, to $\gamma=8$. The solution for $\gamma=1.05$ gives the nonrelativistic limit of the system (1.2)-(1.4) with a good accuracy.

In Fig. 1 the density of the electron current versus time is shown. The quantity $j_e(t)$ normalized by $j_{e0}$ is the current density across the diode with no ions. The values of $j_{e0}$ for various $\gamma$ can be found in [1]. The current is not monotonic in time if the solution reaches the stationary value corresponding to a steady ion flow. The highest current density is observed at the instant at which the forefront of the ions reaches the cathode.

In Figs. 2 and 3 the coordinates $z_f$ of the forefront versus time as well as the density distribution of ions within the diode gap for several time instants [Fig. 3 corresponds to $\gamma=3$; 1) $t=0.54$; 2) $t=0.80$; 3) $t=0.96$; 4) $t=1.17$; 5) $t=4.50$] are given. These results are of value in that they can be compared with the predictions of a simple model describing the initial stage of the diode being cut off by ions [4]. If the ions have not travelled far from the anode, that is, if $1-z_f \ll 1$, then it can be assumed that the field at the point $z_f$ is approximately equal to the electric field $E_a$ at the anode surface in a diode with no ions (the effect of the ion charge on the motion of the electrons in the diode being ignored). This indicates that the ions at the forefront are accelerated by the constant field $E_a$, that is, $1-z_f=E_a t^2/2$. It can be seen that in such a model the ion density is uniform over the interval $z_f < z < 1$, and by virtue of (1.6) it is equal to $n_i = E_a/(1-z_f) = 2/t^2$. One can see from Fig. 3 that for $z > z_f$ the $n_i$ is, in fact, independent of z (with the exception of the region of close proximity to the anode), and this takes place not only for z close to unity, but also for small values of $z_f$. The same can also be observed in the variants with $\gamma = 1.05$; 8. A comparison with the results of numerical computations shows that the obtained functions $z_f(t)$, $n_i(t)$ (which strictly speaking are only valid for $1-z_f \ll 1$) are also fully valid with a good accuracy right up to the instant at which the ions arrive at the cathode. For example, for $\gamma = 3$ one has $E_a = 1.46$ and in accordance with the previous formulas the first ions should reach the cathode at $t_*=1.17$ $(z_f(t_*)=0)$ and at this instant the density of the uniform part of the ions is $n_i = 1.46$. Exact calculations (see Figs. 2 and 3) yield somewhat differing results, namely, $t_* = 1.16$, $n_i = 1.67$.

Finally, in Figs. 4 and 5 the ion energy $W_i$ and the density of the ion current at the cathode surface are shown as a function of time. The latter is denoted by $\tilde{j}_i$ after normalization by the coefficient $j_{e0}[(\gamma+1)m/2M]^{1/2}$. With such an adopted unit of measurement $\tilde{j}_i$ approaches for large t the same limit as $j_e(t)/j_{e0}$. It is noted that the maximal ion energy arriving at the cathode is appreciably (~1.3 times) greater than the accelerating voltage in the diode.

It is obvious that qualitatively all these functions hardly differ for various $\gamma$. A similar situation also arises in computations with two-component anode plasma. Therefore, the computation results given below are only for $\gamma=3$. The ratio of the mass of a heavy ion to that of a light ion was adopted as $\mu = M_h/M_l = 27$ (hydrogen plasma with admixtures of aluminum). The subscripts h and $l$ refer to the heavy and the light components, respectively. In Fig. 6 the density of the electron current flowing through the diode [and in Figs. 7 and 8 the current of the heavy and light ions at the cathode surface (the latter in units of $j_{e0}[(\gamma+1)m/2M_l]^{1/2}$) is shown as a function of time. The computations were carried out for three different values of the heavy-ion concentration: $\alpha=0.1$; 0.5; 0.8 (solid, dash, dashed-dot lines, respectively).

Using the inequality $\mu \gg 1$ we shall assess the conditions when the effect of the heavy ions on the diode operation can be ignored. To this end it is essential that their charge density $n_{hi} \sim j_{hi}/e\ (e\varphi_0/M_l)^{1/2}$ be small compared with $n_{li} \sim j_{li}/e(e\varphi_0/M_l)^{1/2}$. By taking into account that $j_{hi} \sim \alpha j_{li}$, one finds that $\alpha \lessdot \mu^{-1/2}$. In our actual computations one had $\mu^{-1/2} \simeq 0.2$ and therefore this situation can be illustrated by the variant with $\alpha=0.1$. However, even in this case the effect of the heavy component resulting in lower $j_e$ for $t=3$-7 is quite appreciable (see the corresponding curves in Figs. 6 and 1).

In the case of $\alpha \gtrdot \mu^{-1/2}$ the character of the diode operation is quite different. Now the effect of the heavy ions can be neglected only in the initial stage of the process. Indeed, from the start of the ion emission during the time of order $\tau_l$ the light ions cut off the diode gap and the current density becomes $j_{li} \sim j_{e0}(m/M_l)^{1/2}$. The current density of the heavy ions is equal to $j_{hi} \sim \alpha j_{li}$ and during the time interval $\Delta t \sim \tau_l/\alpha$ their charge becomes comparable to the electron charge in the diode gap. Since $\Delta t < \tau_h$, during this time the heavy ions will not succeed in moving far from the anode. The arising charged layer of heavy ions provides a shield for the electric field on the anode* and $j_{hi}$ is reduced to the value of $j_{hi} \sim j_{e0}(m/M_h)^{1/2}$. Correspondingly, the current of light ions is also reduced ($j_{li} \sim j_{0e}(m/M_h)^{1/2}/\alpha$) and this leads to the reduction of $j_e$ to the initial value $j_{e0}$. Only as the diode gap is being filled by heavy ions and the electric charge is neutralized by them does $j_e$ increase (the characteristic time now is $\tau_h$) to the level corresponding to the stationary solution with ion flows.

The above considerations are illustrated by computation results for variants with $\alpha = 0.5; 0.8$. Since now $\alpha \sim 1$, $\Delta t \sim \tau_l$, the effects characteristic for the initial stage of the process appear in the diagrams as splashes $j_e$ and $j_{li}$ of duration $\sim \tau_l$. A further slow change of $j_e$ (see Fig. 6) is due to the motion of heavy ions.

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*Here it may be essential to take into account the thermal expansion of the plasma. If the expansion rate is sufficiently high, then the plasma may cover a thin layer of heavy ions neutralizing their charge. Therefore, the described pattern takes place only when the anode plasma is sufficiently cool.

# NUMERICAL SIMULATION OF THE SELF-FOCUSING OF WAVE PACKETS IN A MEDIUM WITH STRICTION NONLINEARITY

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UDC 535+534.222

During the propagation of powerful laser pulses in many media (e.g., crystals and plasma), the nonlinear increment to the dielectric constant associated with the development of sound perturbations may be very considerable. Striction nonlinearity may lead to the self-focusing of laser pulses, which in turn may be accompanied by the development of severe elastic stresses in crystals.

In this paper we shall make a numerical study of the propagation of axially symmetric wave packets in a medium with striction nonlinearity within the framework of the equations [1, 2]

$$\begin{gathered} i(u_t + vu_z) + \Delta_\perp u + \sigma\rho u = 0, \\ \rho_{tt} - c_s^2\Delta\rho = -\Delta|u|^2 \end{gathered} \tag{1}$$

and the natural boundary conditions

$$\begin{gathered} \partial u/\partial r|_{r=0} = \partial\rho/\partial r|_{r=0} = 0, \\ u(r=\infty) = \rho(r=\infty) = 0, \\ u(|z|=\infty) = \rho(|z|=\infty) = 0, \end{gathered}$$

where u is the envelope of the wave packet; v and $c_s$ are the group velocity of light and the velocity of sound in

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